

# A consistent picture for large penguins in $D \rightarrow \pi^+\pi^-, K^+K^-$

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## Abstract

A long-standing puzzle in charm physics is the large difference between the  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$  decay rates. Recently, the LHCb and CDF collaborations reported a surprisingly large difference between the direct CP asymmetries,  $\Delta\mathcal{A}_{CP}$ , in these two modes. We show that the two puzzles are naturally related in the Standard Model via  $s$ - and  $d$ -quark “penguin contractions”. Their sum gives rise to  $\Delta\mathcal{A}_{CP}$ , while their difference contributes to the two branching ratios with opposite sign. Assuming nominal SU(3) breaking, a U-spin fit to the  $D^0 \rightarrow K^+\pi^-, \pi^+K^-, \pi^+\pi^-, K^+K^-$  decay rates yields large penguin contractions that naturally explain  $\Delta\mathcal{A}_{CP}$ . Expectations for the individual CP asymmetries are also discussed.

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## I. INTRODUCTION

There are several surprising experimental facts in  $D^0$  decays to pairs of charged pseudoscalars. The first one is the long-standing puzzle of the large rate difference between the singly Cabibbo-suppressed (SCS) decays,  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$ . The two amplitudes would be equal in the U-spin symmetric limit, whereas the measured rates yield

$$\left| \frac{\mathcal{A}(D^0 \rightarrow K^+K^-)}{\mathcal{A}(D^0 \rightarrow \pi^+\pi^-)} \right| - 1 = (0.82 \pm 0.02), \quad (1)$$

for their CP averaged magnitudes, after accounting for phase space.<sup>1</sup> This has led to speculation that U-spin breaking in SCS  $D$  decays is  $\mathcal{O}(1)$  [1–8], rather than of the nominal size characterized by

$$\epsilon_U \sim (f_K/f_\pi - 1) \sim \mathcal{O}(0.2). \quad (2)$$

However, such a conclusion is premature, as indicated by the following interesting experimental observations:

1. The CP averaged magnitudes for the Cabibbo-favored (CF)  $D^0 \rightarrow K^-\pi^+$  and doubly Cabibbo-suppressed (DCS)  $D^0 \rightarrow K^+\pi^-$  amplitudes satisfy

$$\left| \frac{V_{cs}V_{ud}}{V_{cd}V_{us}} \frac{\mathcal{A}(D^0 \rightarrow K^+\pi^-)}{\mathcal{A}(D^0 \rightarrow K^-\pi^+)} \right| - 1 = (15.1 \pm 2.8)\%. \quad (3)$$

2. The CP averaged amplitudes satisfy the experimental “sum-rule” relation (a similar sum rule has been discussed in [9])

$$\Sigma_{\text{sum-rule}} = \frac{|\mathcal{A}(D^0 \rightarrow K^+K^-)/V_{cs}V_{us}| + |\mathcal{A}(D^0 \rightarrow \pi^+\pi^-)/V_{cd}V_{ud}|}{|\mathcal{A}(D^0 \rightarrow K^+\pi^-)/V_{cd}V_{us}| + |\mathcal{A}(D^0 \rightarrow K^-\pi^+)/V_{cs}V_{ud}|} - 1 = (4.0 \pm 1.6)\%. \quad (4)$$

The expressions in (3) and (4) would vanish in the U-spin limit. Thus, the fact that they are small experimentally suggests that U-spin is a good approximate symmetry in these decays. The alternative is that U-spin breaking is  $\mathcal{O}(1)$  in the SCS decays under consideration, and nominal in the CF/DCS ones. However, such a hierarchy of U-spin breakings would be left unexplained.

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<sup>1</sup> We define  $|\mathcal{A}(D^0 \rightarrow PP)| \equiv [\bar{\Gamma}(D^0 \rightarrow PP) 8\pi m_D^2/p_c]^{1/2}/(1 \text{ keV})$ , where  $\bar{\Gamma}$  is the CP averaged decay rate and  $p_c$  is the center-of-mass momentum of the final state mesons.

Another interesting result is the surprisingly large time-integrated CP asymmetry difference,  $\Delta\mathcal{A}_{CP} \equiv \mathcal{A}_{CP}(D^0 \rightarrow K^+K^-) - \mathcal{A}_{CP}(D^0 \rightarrow \pi^+\pi^-)$ , recently measured by the LHCb and CDF collaborations [10, 11]. Inclusion of the Babar and Belle measurements of the individual  $K^+K^-$  and  $\pi^+\pi^-$  time-integrated CP asymmetries [12, 13] and the indirect CP asymmetry  $A_r$  [14, 15] yields the world average for the direct CP asymmetry difference [11]

$$\Delta\mathcal{A}_{CP}^{\text{dir}} \equiv \mathcal{A}_{CP}^{\text{dir}}(D \rightarrow K^+K^-) - \mathcal{A}_{CP}^{\text{dir}}(D \rightarrow \pi^+\pi^-) = (-0.67 \pm 0.16) \%. \quad (5)$$

In the Standard Model (SM), the ratio of penguin-to-tree amplitudes is naively of  $\mathcal{O}([V_{cb}V_{ub}/V_{cs}V_{us}]\alpha_s/\pi) \sim 10^{-4}$ , yielding  $\Delta\mathcal{A}_{CP}^{\text{dir}} < 0.1\%$ . This expectation is based on estimates of the “short-distance” penguins with  $b$ -quarks in the loops. While  $\Delta\mathcal{A}_{CP}^{\text{dir}}$  could be a signal of new physics [8, 16–23], it was argued long ago that long-distance effects could conceivably give large direct CP violation (CPV) due to hadronic enhancement of penguin amplitudes [24]. Small CPV was subsequently predicted in [25] using a model for final-state interactions. Making use of lessons learned from the  $D \rightarrow PP$  data that has since become available, it was estimated in [26] (see also [27]) that formally power-suppressed long-distance  $s$ - and  $d$ -quark “penguin contractions” can yield penguin-to-tree ratios of  $\mathcal{O}(0.1\%)$ , thus potentially explaining the observed  $\Delta\mathcal{A}_{CP}^{\text{dir}}$ . More recent works argue that a SM explanation is either marginal [28], or not possible [29].

In this paper we show that the possibility of a large penguin amplitude in the SM acquires further support from the experimental data. A consistent picture emerges in which

1. U-spin breaking is nominal. This helps explaining (3) and (4) with minimal tuning of strong phases.
2. The U-spin invariant sum of the  $s$ - and  $d$ -quark penguin contractions enhances the penguin amplitude, thus explaining  $\Delta\mathcal{A}_{CP}^{\text{dir}}$  in (5).
3. The difference between the  $s$ - and  $d$ -quark penguin contractions, which we refer to as the “broken penguin” (with respect to U-spin), explains the difference in the decay amplitudes in (1).

The last point above requires a broken penguin amplitude that is of the same order as the tree amplitude (with a  $\sim 50\%$  smaller value preferred), yielding substantial interference between the two (see also [7]). In turn, nominal U-spin breaking implies that the penguin

amplitude is enhanced relative to the tree amplitude by  $\mathcal{O}(1/\epsilon_U)$  (with  $\sim 0.5/\epsilon_U$  preferred). This is in the favored range to explain the observed  $\Delta\mathcal{A}_{CP}^{\text{dir}}$ .

The situation we describe in this paper resembles the one which arises in kaon decays: the apparently large isospin breaking in  $K \rightarrow \pi\pi$  [30] results from a combination of nominal isospin breaking and the “ $\Delta I = 1/2$  rule”, i.e., the enhancement of the  $A_0$  amplitude relative to  $A_2$ . Here we suggest that the apparently large U-spin breaking in  $D^0 \rightarrow K^+K^-, \pi^+\pi^-$  decays is a consequence of both nominal U-spin breaking and an enhancement of the  $\Delta U = 0$  penguin matrix elements relative to the tree amplitude.

The paper is organized as follows. In Section II we discuss the relevant  $\Delta C = 1$  effective Hamiltonian, explain our counting in  $\epsilon_U$  for the operator matrix elements, and give the U-spin decomposition of the decay amplitudes. Note that our counting is modified in order to take explicit account of the penguin contractions, thus allowing for their enhancement. Implications of the measured CP averaged decay rates for the penguin contractions are then studied. In Section III we incorporate the CP violation data. We conclude in Section IV. A derivation of the U-spin decomposition together with U-spin breaking is given in Appendix A. Appendix B contains the equivalent “diagrammatic” decomposition of the decay amplitudes. We also provide a translation between the diagrammatic and the U-spin reduced matrix elements.

## II. ENHANCED PENGUINS AND DECAY RATES

The decays

$$\bar{D}^0 \rightarrow K^+\pi^-, \quad \bar{D}^0 \rightarrow K^-\pi^+, \quad \bar{D}^0 \rightarrow \pi^+\pi^-, \quad \bar{D}^0 \rightarrow K^+K^- \quad (6)$$

are related through U-spin. We begin with a discussion of their U-spin decomposition. Using Cabibbo-Kobayashi-Maskawa (CKM) unitarity we can write the Hamiltonian governing SCS decays as

$$H_{\text{eff}}^{\text{SCS}} = \frac{G_F}{\sqrt{2}} \left\{ (V_{cs}V_{us}^* - V_{cd}V_{ud}^*) \sum_{i=1,2} C_i (Q_i^{\bar{s}s} - Q_i^{\bar{d}d}) / 2 \right. \\ \left. - V_{cb}V_{ub}^* \left[ \sum_{i=1,2} C_i (Q_i^{\bar{s}s} + Q_i^{\bar{d}d}) / 2 + \sum_{i=3}^6 C_i Q_i + C_{8g} Q_{8g} \right] \right\} + \text{h.c.}, \quad (7)$$

where

$$Q_1^{\bar{p}p'} = (\bar{p}u)_{V-A}(\bar{c}p')_{V-A}, \quad Q_2^{\bar{p}p'} = (\bar{p}_\alpha u_\beta)_{V-A}(\bar{c}_\beta p'_\alpha)_{V-A} \quad (8)$$

are the “tree operators”,  $Q_{3,\dots,6}$  are the QCD penguin operators, and  $Q_{8g}$  is the chromomagnetic dipole operator. The effective Hamiltonian for Cabibbo-favored (CF) decays contains only the tree operators,

$$H_{\text{eff}}^{\text{CF}} = \frac{G_F}{\sqrt{2}} V_{cs} V_{ud}^* \sum_{i=1,2} C_i Q_i^{\bar{d}s} + \text{h.c.}, \quad (9)$$

and similarly for doubly Cabibbo suppressed (DCS) decays with the replacement  $s \leftrightarrow d$ .

We first work in the limit in which U-spin is exact or nearly so, but with the  $d$  and  $s$  quarks still of two distinguishable flavors. Our working assumption is that the penguin contraction contributions coming from the  $Q_{1,2}$  operators are enhanced compared to the tree amplitude, defined below. The enhancement is parametrized by  $1/\epsilon'$ , where  $\epsilon' \ll 1$ . Thus, at a scale  $\mu \sim m_D$  we have for the  $\mathcal{O}(1/\epsilon')$  U-spin invariant matrix elements

$$\begin{aligned} P \equiv \langle K^+ K^- | \sum_{i=1,2} C_i Q_i^{\bar{d}d} + \sum_{i=3,\dots,8g} C_i Q_i | \bar{D}^0 \rangle &= \\ &= \langle \pi^+ \pi^- | \sum_{i=1,2} C_i Q_i^{\bar{s}s} + \sum_{i=3,\dots,8g} C_i Q_i | \bar{D}^0 \rangle \sim \mathcal{O}(1/\epsilon'), \end{aligned} \quad (10)$$

$$\begin{aligned} T + P \equiv \langle K^+ K^- | \sum_{i=1,2} C_i Q_i^{\bar{s}s} + \sum_{i=3,\dots,8g} C_i Q_i | \bar{D}^0 \rangle &= \\ &= \langle \pi^+ \pi^- | \sum_{i=1,2} C_i Q_i^{\bar{d}d} + \sum_{i=3,\dots,8g} C_i Q_i | \bar{D}^0 \rangle \sim \mathcal{O}(1/\epsilon'). \end{aligned} \quad (11)$$

The  $P$  amplitude is a pure  $\Delta U = 0$  transition. Note that the  $Q_{1,2}$  operators in (10) can only produce the  $K^+ K^-$  final state if the  $\bar{d}d$  quark pairs are contracted (and similarly for  $\bar{s}s$  in the case of  $\pi^+ \pi^-$ ). The contractions are illustrated diagrammatically in Fig. 8 of Appendix B. The contractions also have a non-perturbative field theoretic definition which employs the lattice as a UV regulator. This is what we mean when we refer to the penguin contractions below. In (11) the non-contracted contributions of  $Q_{1,2}$  are part of  $T$ .

The contributions of the penguin operators  $Q_{3,\dots,6}$ ,  $Q_{8g}$  in (10) and (11) are expected to be an order of magnitude smaller than required to explain  $\Delta \mathcal{A}_{CP}^{\text{dir}}$ , see e.g., [26], and are thus ignored throughout this work. Note, however, that the scheme and renormalization-scale dependence in their Wilson coefficients cancels the scheme and scale dependence appearing in the penguin contraction matrix elements of  $Q_{1,2}$  in (10) and (11). This cancelation is understood whenever we refer to  $P$  below.

The  $T$  matrix element is  $\mathcal{O}(1)$  in the  $\epsilon'$  counting. In the U-spin limit it is given by

$$\begin{aligned} T &= -\frac{1}{2} \left( \langle K^+ K^- | \sum_{i=1,2} C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle - \langle \pi^+ \pi^- | \sum_{i=1,2} C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle \right) = \\ &= \langle K^+ \pi^- | \sum_{i=1,2} C_i Q_i^{\bar{d}s} | \bar{D}^0 \rangle = \langle \pi^+ K^- | \sum_{i=1,2} C_i Q_i^{\bar{s}d} | D \rangle \sim \mathcal{O}(1). \end{aligned} \quad (12)$$

This follows from the fact that both  $\langle K^+ \pi^- |$ ,  $(\langle K^+ K^- | - \langle \pi^+ \pi^- |)/\sqrt{2}$ ,  $\langle K^- \pi^+ |$  and  $Q_i^{\bar{d}s}$ ,  $(Q_i^{\bar{s}s} - Q_i^{\bar{d}d})/\sqrt{2}$ ,  $Q_i^{\bar{s}d}$  are U-spin triplets.

The sum of the two amplitudes in the first line of (12) is U-spin breaking, giving a “broken penguin”,

$$P_{\text{break}} \equiv \frac{1}{2} \left( \langle K^+ K^- | \sum_{i=1,2} C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle + \langle \pi^+ \pi^- | \sum_{i=1,2} C_i (Q_i^{\bar{d}d} - Q_i^{\bar{s}s}) | \bar{D}^0 \rangle \right). \quad (13)$$

The leading contribution to  $P_{\text{break}}$  measures the difference between final-state interactions involving the  $\bar{s}s$  and  $\bar{d}d$  contractions. The broken penguin is parametrically of the size

$$P_{\text{break}} \sim \epsilon_U P \sim \mathcal{O}(\epsilon_U/\epsilon') \sim \mathcal{O}(1). \quad (14)$$

In the last equality we have used the scaling  $\epsilon' \sim \epsilon_U$ , which is satisfied by the data, as shown below. For now, however, we keep  $\epsilon'$  and  $\epsilon_U$  separate.

As already stressed, our working assumption is that the matrix elements containing penguin contractions of  $Q_{1,2}$  operators are enhanced. We then have two sets of matrix elements, the ones that are  $\mathcal{O}(1/\epsilon')$  enhanced, and the ones that are not. For each of them there is also an expansion in the U-spin breaking parameter  $\epsilon_U$ . At  $n$ -th order in U-spin breaking the reduced amplitudes that are not enhanced are of  $\mathcal{O}(\epsilon_U^n)$ , while the reduced amplitudes that contain penguin contractions are of  $\mathcal{O}(\epsilon_U^n/\epsilon')$ . For example, summarizing the above results, we have the following scalings

$$T \sim \mathcal{O}(1), \quad P \sim \mathcal{O}(1/\epsilon'), \quad P_{\text{break}} \sim \mathcal{O}(\epsilon_U/\epsilon'). \quad (15)$$

The expressions (12) and (13) are valid to  $\mathcal{O}(1, \epsilon_U/\epsilon')$ . At  $\mathcal{O}(\epsilon_U, \epsilon_U^2/\epsilon')$  the sum of matrix elements in (13) also receives a contribution due to U-spin breaking in  $T$ , changing the l.h.s. from  $P_{\text{break}} \rightarrow P_{\text{break}}(1 - \frac{1}{2}\epsilon_{sd}^{(2)}) + \frac{1}{2}\epsilon_{T1}T$ . In principle, the two contributions – the  $P_{\text{break}} \sim \epsilon_U P$  term and the  $\epsilon_U T$  term – could be separated, if necessary. They correspond to two different topological amplitudes, with the  $\bar{q}q$  fermion fields in the  $Q_{1,2}^{\bar{q}q}$  operators either

contracted or not. In particular, the matrix elements defined in this way could be calculated on the lattice in the (probably not so near) future [31].

The decay amplitudes are derived in Appendices A and B. At order  $\mathcal{O}(\epsilon_U, \epsilon_U^2/\epsilon')$ , and using the notations of Appendix B, they read

$$A(\bar{D}^0 \rightarrow K^+\pi^-) = V_{cs}V_{ud}^*T(1 - \frac{1}{2}\epsilon_{T2}), \quad (16)$$

$$\begin{aligned} A(\bar{D}^0 \rightarrow \pi^+\pi^-) = & -\frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*)(T(1 + \frac{1}{2}\epsilon_{T1}) + P_{\text{break}}(1 - \frac{1}{2}\epsilon_{sd}^{(2)})) \\ & - V_{cb}^*V_{ub}(P(1 - \frac{1}{2}\epsilon_P) + \frac{1}{2}T), \end{aligned} \quad (17)$$

$$\begin{aligned} A(\bar{D}^0 \rightarrow K^+K^-) = & \frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*)(T(1 - \frac{1}{2}\epsilon_{T1}) - P_{\text{break}}(1 + \frac{1}{2}\epsilon_{sd}^{(2)})) \\ & - V_{cb}^*V_{ub}(P(1 + \frac{1}{2}\epsilon_P) + \frac{1}{2}T), \end{aligned} \quad (18)$$

$$A(\bar{D}^0 \rightarrow \pi^+K^-) = V_{cd}V_{us}^*T(1 + \frac{1}{2}\epsilon_{T2}). \quad (19)$$

The coefficients multiplying the U-spin breaking parameters  $\epsilon_i$  are chosen such that typically  $\epsilon_i \sim \mathcal{O}(\epsilon_U)$ . For simplicity the  $V_{cb}^*V_{ub}$  suppressed terms are only given to order  $\mathcal{O}(1, \epsilon_U/\epsilon')$ , i.e. to first subleading order in the expansion. The scaling of the different terms in the  $V_{cb}^*V_{ub}$  suppressed amplitudes is

$$P \sim \mathcal{O}(1/\epsilon'), \quad P\epsilon_P \sim \mathcal{O}(\epsilon_U/\epsilon'), \quad T \sim \mathcal{O}(1). \quad (20)$$

The “tree” parts of the amplitude are also given to the first subleading order in the expansion, which in this case is  $\mathcal{O}(\epsilon_U, \epsilon_U^2/\epsilon')$ , with

$$T \sim \mathcal{O}(1), \quad P_{\text{break}} \sim \mathcal{O}(\epsilon_U/\epsilon'), \quad T\epsilon_{T1}, T\epsilon_{T2} \sim \mathcal{O}(\epsilon_U), \quad P_{\text{break}}\epsilon_{sd}^{(2)} \sim \mathcal{O}(\epsilon_U^2/\epsilon'). \quad (21)$$

We are now ready to check how well the implicitly assumed scaling  $\epsilon' \sim \epsilon_U$  compares with the data. In Fig. 1 we display the result of a fit of (16)-(19) to the measured  $\bar{D}^0 \rightarrow K^+K^-, \pi^+\pi^-, K^\pm\pi^\mp$  branching ratios [32]. Here we can safely neglect the  $V_{cb}^*V_{ub}$ -suppressed contributions. There are three real parameters which are floated in the fit – the magnitudes of  $T$  and  $P_{\text{break}}$  and their relative strong phase. The U-spin breaking parameters  $\epsilon_{T1}$ ,  $\epsilon_{T2}$  and  $\epsilon_{sd}^{(2)}$  are varied in the constrained range  $\epsilon_i \in [0, 0.4]$  for Fig. 1 (left), and in three ranges,  $\epsilon_i \in [0, 0.2]$ ,  $\epsilon_i \in [0, 0.3]$ , and  $\epsilon_i \in [0, 0.4]$  for Fig. 1 (right). The branching ratios can be fit perfectly, so that the  $\chi^2$  global minimum is  $\chi_{\text{min}}^2 = 0$ . The fit shows that

$$P_{\text{break}} \sim T, \quad (22)$$

in accord with our  $\epsilon' \sim \epsilon_U$  counting. This is a direct consequence of the large difference between the  $D \rightarrow K^+K^-$  and  $D \rightarrow \pi^+\pi^-$  decay rates.

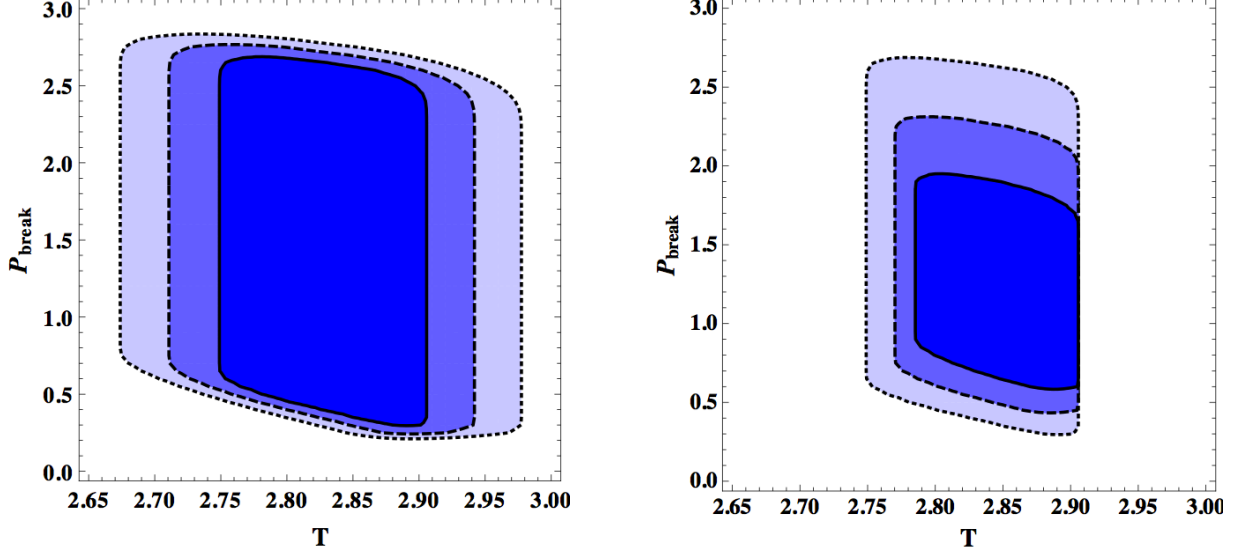


FIG. 1: Constraints on  $P_{\text{break}}$  vs.  $T$  from the fit to the branching ratios. Left: the contours denote regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted). Right: the constraints at  $1\sigma$  but with  $\epsilon_i$  varied in the ranges  $\epsilon_i \in [0, 0.2]$  (solid),  $\epsilon_i \in [0, 0.3]$  (dashed),  $\epsilon_i \in [0, 0.4]$  (dotted).

### III. ENHANCED PENGUINS AND CP VIOLATION

The time-integrated CP asymmetry for SCS  $D^0$  decays to a final CP eigenstate  $f$  is defined as

$$\mathcal{A}_{CP}(D \rightarrow f) \equiv \frac{\Gamma(D \rightarrow f) - \Gamma(\bar{D} \rightarrow f)}{\Gamma(D \rightarrow f) + \Gamma(\bar{D} \rightarrow f)}. \quad (23)$$

It receives both direct and indirect CP violation contributions (see, for example, [16]). In the SM the indirect CP violation lies well below the present experimental sensitivity. We therefore assume that within the SM the measurement of  $\mathcal{A}_{CP}(D \rightarrow f)$  equals the direct CP asymmetry

$$\mathcal{A}_f^{\text{dir}} \equiv \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} = 2r_f \sin \gamma \sin \delta_f. \quad (24)$$

Here we have used the fact that in the SM the CP-conjugate decay amplitudes for CP even final states can be written as

$$\begin{aligned} A_f &\equiv A(D \rightarrow f) = A_f^T [1 + r_f e^{i(\delta_f - \gamma)}], \\ \bar{A}_f &\equiv A(\bar{D} \rightarrow f) = A_f^T [1 + r_f e^{i(\delta_f + \gamma)}]. \end{aligned} \quad (25)$$

$A_f^T$  is the dominant amplitude that is proportional to  $(V_{cs}V_{us}^* - V_{cd}V_{ud}^*)$ , see (7), and  $r_f$  is the relative magnitude of the subleading amplitude, which is proportional to  $V_{cb}V_{ub}^*$ . It carries the weak CKM phase  $\gamma = (67.3_{-3.5}^{+4.2})^\circ$  [33] and the relative strong phase  $\delta_f$ .



We perform a fit to the branching ratios and CP asymmetries to determine  $r_f$  for  $f = K^+K^-$ ,  $\pi^+\pi^-$ . In the fit we use the HFAG averages for the individual time-integrated CP asymmetries<sup>2</sup> (which includes the Babar [12], Belle [13], and CDF measurements [34]),

$$\begin{aligned}\mathcal{A}_{CP}(D \rightarrow \pi^+\pi^-) &= (0.22 \pm 0.24 \pm 0.11)\%, \\ \mathcal{A}_{CP}(D \rightarrow K^+K^-) &= (-0.24 \pm 0.22 \pm 0.10)\%,\end{aligned}\tag{26}$$

and their difference measured at LHCb [10]

$$\Delta\mathcal{A}_{CP} = \mathcal{A}_{CP}(D \rightarrow K^+K^-) - \mathcal{A}_{CP}(D \rightarrow \pi^+\pi^-) = (-0.82 \pm 0.21 \pm 0.11)\%,\tag{27}$$

and CDF [11]

$$\Delta\mathcal{A}_{CP} = (-0.62 \pm 0.21 \pm 0.10)\%.\tag{28}$$

For strong phases  $\delta_f \sim \mathcal{O}(1)$  we have

$$\Delta\mathcal{A}_{CP} \sim 4r_f,\tag{29}$$

using the fact that  $\sin \gamma \sim 0.9$ , and the U-spin based expectation that  $\mathcal{A}_{CP}(D \rightarrow K^+K^-)$  and  $\mathcal{A}_{CP}(D \rightarrow \pi^+\pi^-)$  have opposite signs. In order to explain the central values of  $\Delta\mathcal{A}_{CP}$  one needs

$$r_f \sim 0.2\%,\tag{30}$$

or, equivalently,

$$P/T \sim 3,\tag{31}$$

after accounting for CKM factors. In [26] the penguin contraction contributions were estimated to yield  $r_f \sim 0.1\%$ , (or  $P/T \sim 1.6$ ), with a factor of a few uncertainty. This motivates us to regard a hierarchy for  $P/T$  that is much larger than (31) as unlikely.

Our main point is that under the assumption of nominal U-spin breaking, a broken penguin  $P_{\text{break}}$  which explains the difference of the  $D^0 \rightarrow K^+K^-$  and  $D^0 \rightarrow \pi^+\pi^-$  decay rates implies a  $\Delta U = 0$  penguin  $P$  that naturally yields (30) and the observed  $\Delta\mathcal{A}_{CP}$ . The scaling  $P_{\text{break}} \sim \epsilon_U P$  together with our fit result  $P_{\text{break}} \sim T/2$  (see Fig. 1) yields the estimate

$$r_{\pi^+\pi^-, K^+K^-} \simeq \left| \frac{V_{cb}V_{ub}}{V_{cs}V_{us}} \right| \cdot \left| \frac{P}{T \pm P_{\text{break}}} \right| \sim \frac{|V_{cb}V_{ub}|}{|V_{cs}V_{us}|} \frac{1}{2\epsilon_U} \sim 0.2\%,\tag{32}$$

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<sup>2</sup> The average makes sense in the limit of negligible indirect CP asymmetry.

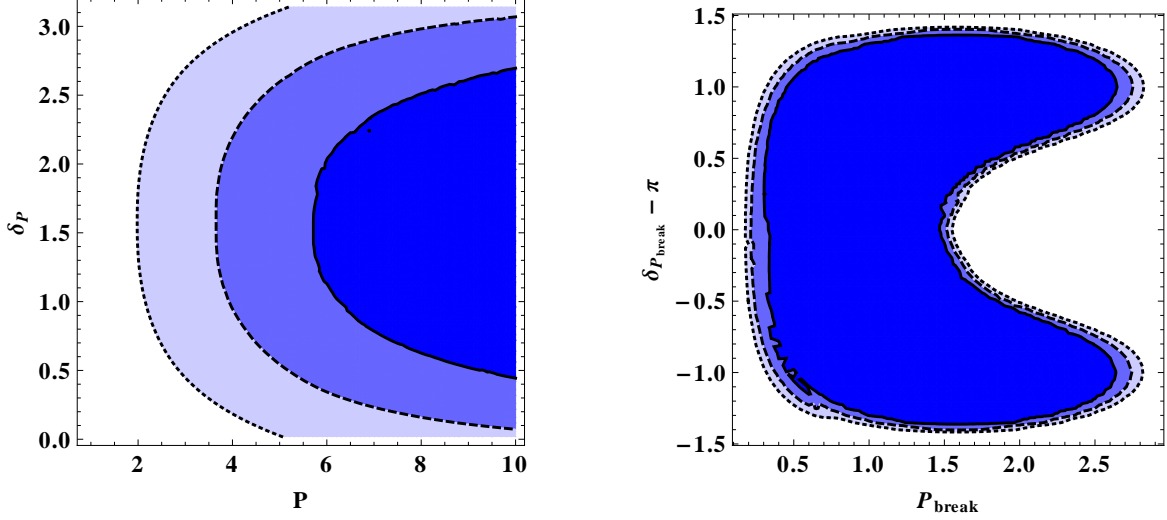


FIG. 2: Constraints on  $\delta_P$  vs.  $P$  (left) and  $\delta_{P_{\text{break}}}$  vs.  $P_{\text{break}}$  (right) following from the fit to branching ratios and CP asymmetries at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

for  $\epsilon_U \sim 0.2$ , consistent with (30).

In order to exhibit this result in detail, we fit the expressions in Eqs. (16)–(19) to the four branching ratios and the time-integrated CP asymmetries in (26)–(28). The latter are identified with the corresponding direct CP asymmetries, under our assumption of negligible indirect CP violation in the SM. This gives us eight measurements that are fitted using five unconstrained real parameters: the magnitudes of  $T, P, P_{\text{break}}$  and the two relative strong phases. In addition there are four U-spin breaking parameters,  $\epsilon_{T1}, \epsilon_{T2}, \epsilon_{sd}^{(2)}, \epsilon_P$  that enter at the first subleading order. They are allowed to lie in the range  $[0, 0.4]$  with arbitrary strong phases. The  $\chi^2$  global minimum is  $\chi_{\text{min}}^2 = 1.14$ . It is not zero because the measurements of  $\Delta\mathcal{A}_{CP}$ ,  $\mathcal{A}_{CP}(K^+K^-)$ , and  $\mathcal{A}_{CP}(\pi^+\pi^-)$  are only consistent at the  $\sim 1\sigma$  level.

In Fig. 2 (left) we show the constraints on  $P$  vs. the strong phase  $\delta_P \equiv \arg(P/T)$  obtained from the fit. As expected, small values of the penguin amplitude  $P$  require a strong phase close to  $\pi/2$ , whereas larger values of  $P$  allow for smaller phases. It is important to note that the minimum value of  $P$  required at  $1\sigma$  is  $\approx 5.8$ , or roughly a factor of 2 larger than  $T_{\text{avg}} = 2.83$ , the average value of  $T$  in our normalization, which can be read off of Fig. 1. We also note that significant strong phases are typical for  $P_{\text{break}}$ , as shown in Fig. 2 (right), where constraints on  $P_{\text{break}}$  vs. the strong phase difference  $\delta_{P_{\text{break}}} \equiv \arg(P_{\text{break}}/T)$  are shown.

Our main results are contained in Fig. 3 and Fig. 4. We introduce the parameter  $\epsilon_{sd}^{(1)}$ ,

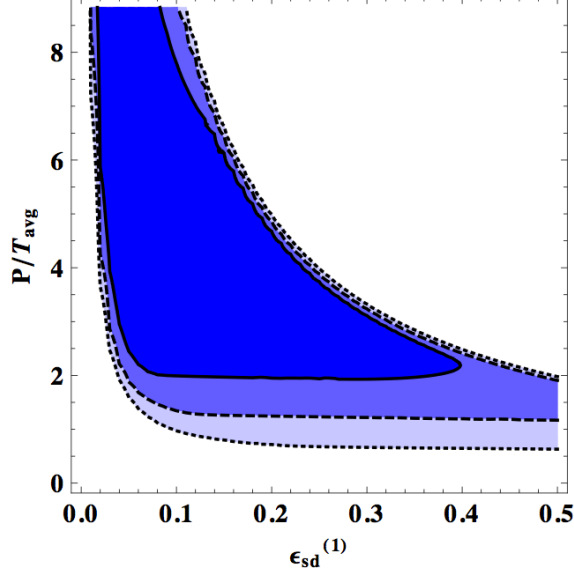


FIG. 3: Constraints on  $P/T_{\text{avg}}$  vs.  $\epsilon_{sd}^{(1)}$ , where  $T_{\text{avg}} = 2.83$ . The contours denote regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

such that

$$P_{\text{break}} = \epsilon_{sd}^{(1)} P, \quad (33)$$

as in (B12). If our fit favored  $\epsilon_{sd}^{(1)}$  in the nominal range for U-spin breaking, it would support large penguins and a SM explanation for  $\Delta\mathcal{A}_{CP}$ , as in (32).

In Fig. 3 the results for  $P/T_{\text{avg}}$  vs.  $\epsilon_{sd}^{(1)}$  are shown for an extended range,  $P \leq 25$ , although it is understood that the range  $P \lesssim 10$  is physically preferred. Indeed, at  $1\sigma$  we find that  $\epsilon_{sd}^{(1)}$  naturally falls into the nominal range  $[0.1, 0.3]$ , for the physically reasonable range  $P/T_{\text{avg}} \lesssim 3.5$ , equivalent to  $P \lesssim 10$ . Note that the lower edges of  $P/T_{\text{avg}}$  correspond to the minimal values of  $P$  seen in Fig. 2. Higher values of  $P$  are compensated by smaller values of  $\epsilon_{sd}^{(1)}$  to yield the observed difference between the  $K^+K^-$  and  $\pi^+\pi^-$  rates, and by smaller strong phases to yield the observed CP asymmetries.

Fig. 4 directly addresses the question of whether we can accommodate the CP asymmetries with nominal U-spin breaking. In Fig. 4 we show the values for  $\Delta\mathcal{A}_{CP}$  for the allowed regions in Fig. 3, for the physically more motivated range  $P \lesssim 10$ , and for an extended range of  $P$ . We see that we can naturally explain the world average for  $\Delta\mathcal{A}_{CP} = -0.67 \pm 0.16$  in (5) with nominal U-spin breaking. Note in particular that, while values of  $P > 10$  allow for marginally larger absolute values for  $\Delta\mathcal{A}_{CP}$ , the lower bound on  $P$  arising from the need to explain the difference of the branching ratios translates into a lower bound on the magnitude

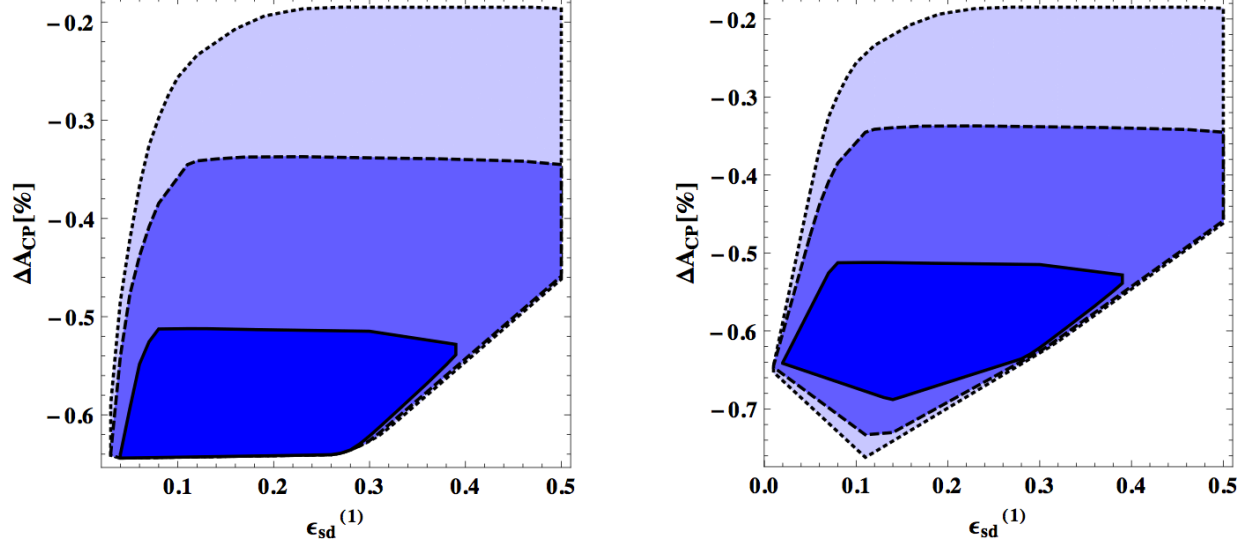


FIG. 4: Allowed values of  $\Delta\mathcal{A}_{CP}$  vs.  $\epsilon_{sd}^{(1)}$  corresponding to the regions shown in Fig. 3 for  $P \leq 10$  (left) and  $P \leq 25$  (right). The contours denote regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

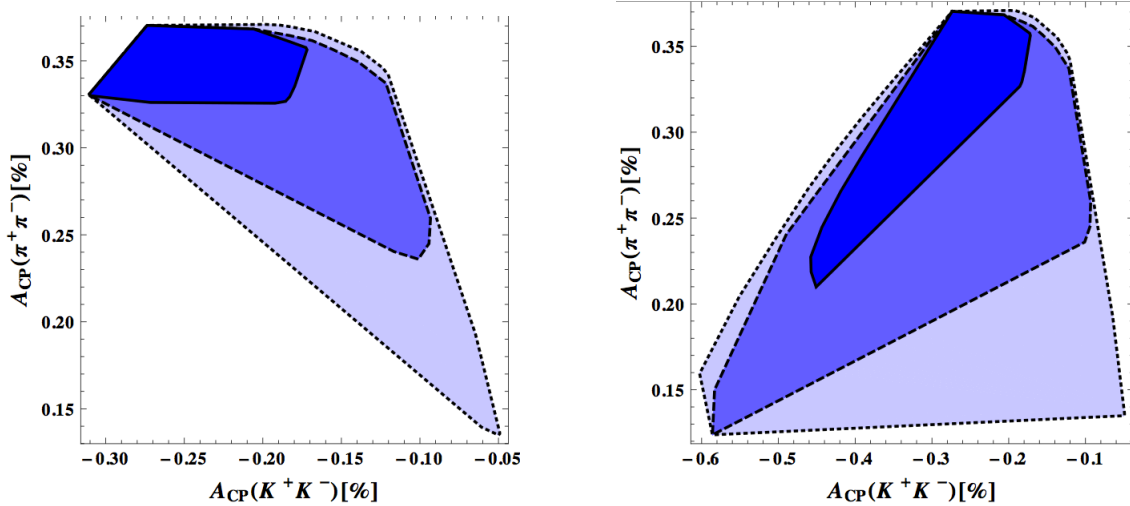


FIG. 5: Result of the fit for  $\mathcal{A}_{CP}(\pi^+\pi^-)$  vs.  $\mathcal{A}_{CP}(K^+K^-)$ , with  $P \leq 10$  (left) and  $P \leq 25$  (right). The contours denote regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

of  $\Delta\mathcal{A}_{CP}$  (the upper edges in Fig. 4). The somewhat unexpectedly large measured value is thus naturally explained.

In Fig. 5 we show the constraints on the individual CP asymmetries,  $\mathcal{A}_{CP}(\pi^+\pi^-)$  and  $\mathcal{A}_{CP}(K^+K^-)$ , that follow from our fit. In the U-spin limit we would have  $\mathcal{A}_{CP}(\pi^+\pi^-) = -\mathcal{A}_{CP}(K^+K^-)$ . For nominal U-spin breaking and to  $\mathcal{O}(\epsilon'/\epsilon_U)$ , we have  $P_{\pi^+\pi^-} = P_{K^+K^-}$ .

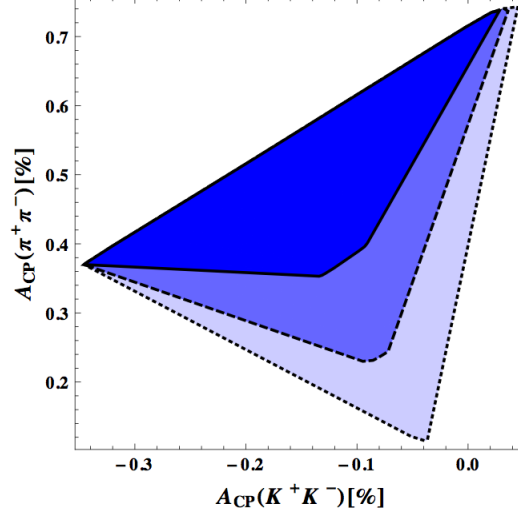


FIG. 6: Result of the fit for  $\mathcal{A}_{CP}(\pi^+\pi^-)$  vs.  $\mathcal{A}_{CP}(K^+K^-)$  without the individual  $CPV$  measurements, for  $P \leq 10$ . The contours denote regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

Thus, we expect the asymmetries to scale like

$$\frac{\mathcal{A}_{CP}(\pi^+\pi^-)}{\mathcal{A}_{CP}(K^+K^-)} \approx - \left| \frac{A(\bar{D}^0 \rightarrow \pi^+\pi^-)}{A(\bar{D}^0 \rightarrow K^+K^-)} \right| \approx -1.8(1 + \mathcal{O}(\epsilon_U)), \quad (34)$$

This is seen to be true for the physically more motivated range  $P \lesssim 10$ , while for larger values of  $P$  much larger values of  $|\mathcal{A}_{CP}(K^+K^-)|$  are still allowed by the data. Although the fit contains the individual CP asymmetry measurements as inputs, this is a non-trivial result given that their  $1\sigma$  intervals are substantially larger than those returned by the fit. For completeness, in Fig. 6 we show the result obtained without inputting the individual CP asymmetry measurements. A larger hierarchy for the individual CP asymmetries becomes possible at  $1\sigma$ .

Finally, let us comment on the sum rule in (4), which is fulfilled experimentally to  $\mathcal{O}(4\%)$ . The corresponding amplitude-level sum rule (absolute values removed) is satisfied to  $\mathcal{O}(\epsilon_U^2)$ . Taking the square roots of the branching ratios at  $\mathcal{O}(\epsilon_U/\epsilon')$ , (4) becomes

$$\Sigma_{\text{sum-rule}} = \left( 1 - \frac{1}{2} \left| 1 - \frac{P_{\text{break}}}{T} \right| - \frac{1}{2} \left| 1 + \frac{P_{\text{break}}}{T} \right| + \mathcal{O}(\epsilon_U, \epsilon_U^2/\epsilon') \right) = \mathcal{O}(4\%). \quad (35)$$

Note that  $\Sigma_{\text{sum-rule}}$  is not necessarily small for  $P_{\text{break}} \sim T$  and large relative strong phase. For instance,  $|P_{\text{break}}| = |T|$  yields a maximum for  $\Sigma_{\text{sum-rule}}$  of 20%, realized at  $\delta_{P_{\text{break}}} = 90^\circ$ . However, the sum-rule decreases rapidly for smaller  $P_{\text{break}}$  and  $\delta_P$ . For example, for the choices  $P_{\text{break}}/T = 0.5$  (or  $P_{\text{break}} \approx 1.4$ ) and  $\delta_{P_{\text{break}}} = \pm 45^\circ$ , which lie near the two “foci” of

the  $1\sigma$  region in Fig. 2 (right), the sum rule is reduced to  $\approx 6.6\%$  at  $\mathcal{O}(\epsilon_U/\epsilon')$ , marginally larger than experiment. We have checked that the degree of tuning in the description of all observables considered is modest, about 1 part in 3, with the dominant effect due to the sum rule. This is what one would expect from the  $\mathcal{O}(\epsilon_U, \epsilon_U^2/\epsilon')$  corrections to (35), assuming U-spin breaking of nominal size. These corrections would necessitate substantial tuning of  $\Sigma_{\text{sum-rule}}$  if  $\epsilon_U$  were large, thus providing further support for nominal U-spin breaking.

#### IV. CONCLUSION

We have shown that the penguin contraction matrix elements of the standard-model operators  $Q_{1,2}$  can provide a consistent picture for large penguins in singly Cabibbo-suppressed  $D \rightarrow PP$  decays. The U-spin violating contractions of  $Q_{1,2}$  explain the long-standing puzzle of a significantly larger branching ratio for  $D \rightarrow K^+K^-$  compared to  $D \rightarrow \pi^+\pi^-$ . At the same time, the U-spin conserving contractions of  $Q_{1,2}$  are of the correct size to naturally explain the large CP asymmetry difference  $\Delta\mathcal{A}_{CP} = \mathcal{A}_{CP}(D \rightarrow K^+K^-) - \mathcal{A}_{CP}(D \rightarrow \pi^+\pi^-)$  measured by LHCb and CDF. A crucial observation, borne out by a detailed U-spin analysis, is that U-spin breaking of nominal size,  $\mathcal{O}(20\%)$ , correctly relates the magnitudes of the two phenomena. On this basis, we conclude that large direct CP asymmetries of order a few per mille are not surprising given the size of  $\text{Br}(D \rightarrow K^+K^-)/\text{Br}(D \rightarrow \pi^+\pi^-)$ .

#### Acknowledgements

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#### Appendix A: Formal U-spin decomposition

In this appendix we perform the U-spin decomposition of the  $D \rightarrow P^+P^-$  decays ( $P = K, \pi$ ), including U-spin breaking. The most general expressions for the amplitudes are

truncated at second order in U-spin breaking, that is, there are no new hadronic matrix elements introduced at higher orders. The U-spin decomposition is written in a form in which it is clear which reduced matrix elements contain the penguin contractions. We assume that these matrix elements are dynamically enhanced. A translation is provided between the amplitudes in the U-spin decomposition and the decomposition given in Section II.

## 1. General discussion

The  $D^0$  meson is a U-spin singlet. The decay operators involve a down-type quark and a down-type antiquark, and thus in general they can be written as a sum of a U-spin triplet and a U-spin singlet. The leading operators (12) form a triplet, while the U-spin singlet operator is proportional to  $V_{cb}V_{ub}^*$ . It is completely negligible as far as the CP averaged rates are concerned, due to the CKM suppression. In terms of tensor notation the two operators are given by

$$H_1 = \langle H_1 \rangle \begin{pmatrix} \frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) & V_{cs}^*V_{ud} \\ V_{cd}V_{us}^* & -\frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) \end{pmatrix}, \quad H_0 = -V_{cb}^*V_{ub} \langle H_0 \rangle \mathbf{I}_{2 \times 2}, \quad (\text{A1})$$

where the  $\langle H_i \rangle$  are hadronic coefficients, and  $H_0$  is proportional to the identity matrix  $\mathbf{I}_{2 \times 2}$ . We also split the final states into a U-spin triplet and singlet

$$M_1 = \begin{pmatrix} \frac{1}{2}(K^+K^- - \pi^+\pi^-) & \pi^+K^- \\ \pi^-K^+ & -\frac{1}{2}(K^+K^- - \pi^+\pi^-) \end{pmatrix}, \quad M_0 = \frac{1}{2}(\pi^+\pi^- + K^+K^-) \mathbf{I}_{2 \times 2}. \quad (\text{A2})$$

The U-spin breaking is induced by the nonzero strange-quark mass term,  $m_s \bar{s}s$ . Subtracting the singlet piece, the breaking introduces a spurion that is a U-spin triplet

$$M_\epsilon = \begin{pmatrix} \epsilon/2 & 0 \\ 0 & -\epsilon/2 \end{pmatrix}, \quad (\text{A3})$$

where  $\epsilon$  is a small parameter which parametrizes U-spin breaking, i.e.,  $\epsilon \sim \epsilon_U$ .

In the U-spin limit there are two reduced matrix elements

$$t_0 \propto \langle f_1 | H_1 | 0 \rangle, \quad p_0 \propto \langle f_0 | H_0 | 0 \rangle, \quad (\text{A4})$$

where  $f_0$  and  $f_1$  are the singlet and triplet states corresponding to  $M_0$  and  $M_1$ , respectively, and  $|0\rangle$  is the U-spin singlet  $D^0$  meson. At  $\mathcal{O}(\epsilon)$  there are three additional reduced matrix

elements

$$s_1 \propto \langle f_0 | (H_1 \times \mathbb{1}_\epsilon)_0 | 0 \rangle, \quad t_1 \propto \langle f_1 | (H_1 \times \mathbb{1}_\epsilon)_1 | 0 \rangle, \quad p_1 \propto \langle (f_1 \times \mathbb{1}_\epsilon)_0 | H_0 | 0 \rangle, \quad (\text{A5})$$

where  $\mathbb{1}_\epsilon$  represents the U-spin breaking spurion  $M_\epsilon$ . At  $\mathcal{O}(\epsilon^2)$  there are three more reduced matrix elements,

$$t_2 \propto \langle (f_1 \times \mathbb{1}_\epsilon)_0 | (H_1 \times \mathbb{1}_\epsilon)_0 | 0 \rangle, \quad t'_2 \propto \langle (f_1 \times \mathbb{1}_\epsilon)_1 | (H_1 \times \mathbb{1}_\epsilon)_1 | 0 \rangle, \quad (\text{A6})$$

$$p_2 \propto \langle f_0 | H_0 \times (\mathbb{1}_\epsilon \times \mathbb{1}_\epsilon)_0 | 0 \rangle.$$

In terms of the tensor notation, we have the identities

$$(H_1 \times \mathbb{1}_\epsilon)_0 \equiv \{H_1, M_\epsilon\} = \epsilon \langle H_1 \rangle \frac{1}{2} (V_{cs} V_{us}^* - V_{cd} V_{ud}^*) \mathbf{I}_{2 \times 2}$$

$$(H_1 \times \mathbb{1}_\epsilon)_1 \equiv [H_1, M_\epsilon] = \epsilon \langle H_1 \rangle \begin{pmatrix} 0 & -V_{cs}^* V_{ud} \\ V_{cd} V_{us}^* & 0 \end{pmatrix}. \quad (\text{A7})$$

Thus, the anticommutator (commutator) of  $H_1$  and  $M_\epsilon$  projects onto a singlet (triplet) operator. Similarly, we have

$$(f_1 \times \mathbb{1}_\epsilon)_0 \equiv \{M_\epsilon, M_1\} = \epsilon \frac{1}{2} (K^+ K^- - \pi^+ \pi^-) \mathbf{I}_{2 \times 2}$$

$$(f_1 \times \mathbb{1}_\epsilon)_1 \equiv [M_\epsilon, M_1] = \epsilon \begin{pmatrix} 0 & \pi^+ K^- \\ -\pi^- K^+ & 0 \end{pmatrix}, \quad (\text{A8})$$

and the anticommutator (commutator) of  $M_1$  and  $M_\epsilon$  projects onto a singlet (triplet) state.

We can now write the two-body decay Hamiltonian to  $\mathcal{O}(\epsilon^2)$  as

$$\mathcal{H}_1 = t_0 \text{Tr}(H_1 M_1) + \frac{1}{2} t_1 \text{Tr}([H_1, M_\epsilon] M_1) + \frac{1}{4} t_2 \text{Tr}(\{H_1, M_\epsilon\} \{M_\epsilon, M_1\})$$

$$+ \frac{1}{4} t'_2 \text{Tr}([H_1, M_\epsilon] [M_\epsilon, M_1]) + s_1 \text{Tr}(\{H_1, M_\epsilon\} M_0), \quad (\text{A9})$$

for the U-spin triplet operator, and

$$\mathcal{H}_0 = p_0 \text{Tr}(H_0 M_0) + \frac{1}{2} p_1 \text{Tr}(H_0 \{M_\epsilon, M_1\}) + p_2 \text{Tr}(H_0 M_\epsilon^2 M_0). \quad (\text{A10})$$

for the U-spin singlet operator.

Choosing a convenient final state phase convention, the decay amplitudes can be read off from (A9), (A10), yielding

$$A(\bar{D}^0 \rightarrow K^+ \pi^-) = V_{cs} V_{ud}^* (t_0 - \frac{1}{2} t_1 \epsilon + \frac{1}{4} t'_2 \epsilon^2),$$

$$A(\bar{D}^0 \rightarrow \pi^+ \pi^-) = -\frac{1}{2} (V_{cs} V_{us}^* - V_{cd} V_{ud}^*) (t_0 + s_1 \epsilon + \frac{1}{2} t_2 \epsilon^2) - V_{cb} V_{ub}^* (p_0 - \frac{1}{2} p_1 \epsilon + \frac{1}{4} p_2 \epsilon^2),$$

$$A(\bar{D}^0 \rightarrow K^+ K^-) = \frac{1}{2} (V_{cs} V_{us}^* - V_{cd} V_{ud}^*) (t_0 - s_1 \epsilon + \frac{1}{2} t_2 \epsilon^2) - V_{cb} V_{ub}^* (p_0 + \frac{1}{2} p_1 \epsilon + \frac{1}{4} p_2 \epsilon^2),$$

$$A(\bar{D}^0 \rightarrow \pi^+ K^-) = V_{cd} V_{us}^* (t_0 + \frac{1}{2} t_1 \epsilon + \frac{1}{4} t'_2 \epsilon^2), \quad (\text{A11})$$



where we have made the replacements  $t_i \langle H_1 \rangle \rightarrow t_i$  and  $p_i \langle H_0 \rangle \rightarrow p_i$  for the reduced matrix elements.

The U-spin expansion is just a basis rotation, so that in general there can only be six independent decay amplitudes, four associated with decays to the triplet final state  $f_1$  and two with decays to the singlet final state  $f_0$ . Thus, of the three reduced matrix elements introduced at  $\mathcal{O}(\epsilon^2)$ , only one combination is a new linearly independent amplitude. In fact, we can see directly from the above amplitude expressions that  $p_2$  can be absorbed into  $p_0$ , and that one linear combination of  $t_2$  and  $t'_2$  can be absorbed into  $t_0$ , leaving another linear combination of the two as the new linearly independent amplitude.

There are a number of conclusions that one can draw from the above decomposition at different orders in  $\epsilon$ . In the U-spin symmetric limit we have the following known relations: (i) all 4 decay rates are equal (up to CKM prefactors); and (ii) the direct CP asymmetries in the SCS decays are equal in magnitude and opposite in sign.

Working to  $\mathcal{O}(\epsilon)$ , and neglecting the small terms proportional to  $V_{cb}V_{ub}^*$ , the four amplitudes depend on three reduced matrix elements. Thus, there is one relation among the  $\bar{D}^0$  decay amplitudes, which is given by

$$\frac{\bar{A}_{K^-\pi^+}}{V_{cs}V_{ud}^*} + \frac{\bar{A}_{K^+\pi^-}}{V_{cd}V_{us}^*} = \frac{\bar{A}_{K^+K^-}}{V_{cs}V_{us}^*} + \frac{\bar{A}_{\pi^+\pi^-}}{V_{cd}V_{ud}^*}, \quad (\text{A12})$$

and similarly for the CP conjugate decays. This sum rule is broken at  $\mathcal{O}(\epsilon^2)$ , whereas the individual amplitudes are  $\mathcal{O}(1)$ . The experimental relation (4) is also satisfied to first order, that is,  $\Sigma_{\text{sum-rule}} = \mathcal{O}(\epsilon^2)$ .

The rate difference between the  $K^+\pi^-$  and  $K^-\pi^+$  modes and the rate difference between the  $K^+K^-$  and  $\pi^+\pi^-$  modes arise at order  $\mathcal{O}(\epsilon)$ . However, the latter is observed to be  $\mathcal{O}(1)$ , while the former is small. The immediate conclusion is that

$$s_1\epsilon \gg t_1\epsilon. \quad (\text{A13})$$

There are two ways in which this relation could be realized. The first one is that there is a U-spin breaking hierarchy which remains unexplained, i.e., a hierarchy of  $\epsilon$ 's, such that the breaking is much larger for  $s_1$  than for  $t_1$ . The second possibility, which is the one we have pursued in this paper, is that all U-spin breaking is of nominal size, but that  $s_1$  is enhanced relative to  $t_1$  due to the penguin contractions.

## 2. The penguin contractions

Our working assumption is that at a given order in  $\epsilon$  the contractions of the  $s\bar{s}$  and  $d\bar{d}$  fields give the dominant effects. The reduced matrix elements to which the contractions contribute are the ones in which both the transition Hamiltonian and the final state can be written as U-spin singlets. Thus, they are identified with the following traces

$$\text{Tr}[(M_0 \text{ or } \{M_\epsilon, M_1\}) \times (H_0 \text{ or } \{H_1, M_\epsilon\})]. \quad (\text{A14})$$

According to (A9) and (A10), these are the  $H_0$  matrix elements  $p_0, p_1, p_2$  and the  $H_1$  matrix elements  $s_1, t_2$ . We elaborate below, and check the consequences for the U-spin decomposition.

At  $\mathcal{O}(\epsilon^0)$ ,  $H_0$  gives rise to the reduced matrix element  $p_0$ . This involves matrix elements of  $Q_{1,2}^{\bar{s}s} + Q_{1,2}^{\bar{d}d}$  for the singlet final state  $(|K^+K^- \rangle + |\pi^+\pi^- \rangle)/\sqrt{2}$ , see (A1) and (7). Therefore,  $p_0$  contains both contracted and non-contracted contributions. Also at  $\mathcal{O}(\epsilon^0)$ , the  $U_3 = 0$  component of  $H_1$  gives rise to the reduced matrix element  $t_0$ . It involves matrix elements of  $Q_{1,2}^{\bar{s}s} - Q_{1,2}^{\bar{d}d}$  for the triplet final state  $(|K^+K^- \rangle - |\pi^+\pi^- \rangle)/\sqrt{2}$ . Therefore, the  $s\bar{s}$  and  $d\bar{d}$  contractions must cancel in  $t_0$  at  $\mathcal{O}(\epsilon^0)$ .

To account for the dominance of the contractions we introduce a second small parameter  $\epsilon'$ , such that any matrix element to which they contribute is enhanced by  $\mathcal{O}(1/\epsilon')$ . Without loss of generality, we can define it as

$$\epsilon' \equiv \left| \frac{t_0}{p_0} \right|. \quad (\text{A15})$$

Thus, in  $p_0$  the ratio of non-contracted to contracted contributions is also of  $\mathcal{O}(\epsilon')$ .

The contractions enter the matrix elements of  $H_1$  for the first time at  $\mathcal{O}(\epsilon)$ . This happens for the  $s_1$  matrix element for which the final state is a singlet. This is because the operator  $(H_1 \times \mathbb{1}_\epsilon)_0 \equiv \{H_1, M_\epsilon\}$  contains the sum  $Q_{1,2}^{\bar{s}s} + Q_{1,2}^{\bar{d}d}$ , see (A7) and (13). Thus, the contractions do not cancel, but add up. This is also the case for the contractions in  $p_1$  at  $\mathcal{O}(\epsilon)$ , and the contractions in  $p_2$  and  $t_2$  at  $\mathcal{O}(\epsilon^2)$ . Note, however, that  $p_1$  and  $t_2$  involve the  $U_3 = 0$  triplet final state. We can transform it to a singlet with the aid of the U-spin breaking spurion i.e.,  $\{M_\epsilon, M_0\}$ . We are led to the following  $\epsilon'$  counting rule: a reduced matrix element that is associated with one of the traces in (A14) is  $\mathcal{O}(1/\epsilon')$ , and is  $\mathcal{O}(1)$  otherwise. Explicitly,

$$s_1 \sim t_2 \sim p_{0,1,2} \sim \mathcal{O}(1/\epsilon'), \quad t_{0,1} \sim t'_2 \sim \mathcal{O}(1). \quad (\text{A16})$$

Taking  $\epsilon' \sim \epsilon \sim \epsilon_U$ , the different amplitudes are thus

$$\begin{aligned} p_0 &\sim \mathcal{O}(1/\epsilon), & t_0 &\sim s_1\epsilon \sim p_1\epsilon \sim \mathcal{O}(1), \\ t_1\epsilon &\sim t_2\epsilon^2 \sim p_2\epsilon^2 \sim \mathcal{O}(\epsilon), & t'_2\epsilon^2 &\sim \mathcal{O}(\epsilon^2). \end{aligned} \quad (\text{A17})$$

Keeping only terms to order  $\mathcal{O}(\epsilon) \sim \mathcal{O}(\epsilon^2/\epsilon') \sim \mathcal{O}(\epsilon_U)$ , i.e. the leading and subleading terms for both CKM structures, we finally have

$$\begin{aligned} A(\bar{D}^0 \rightarrow K^+\pi^-) &= V_{cs}V_{ud}^* (t_0 - \tfrac{1}{2}t_1\epsilon), \\ A(\bar{D}^0 \rightarrow \pi^+\pi^-) &= -\tfrac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) (t_0 + s_1\epsilon + \tfrac{1}{2}t_2\epsilon^2) - V_{cb}V_{ub}^* (p_0 - \tfrac{1}{2}p_1\epsilon), \\ A(\bar{D}^0 \rightarrow K^+K^-) &= \tfrac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) (t_0 - s_1\epsilon + \tfrac{1}{2}t_2\epsilon^2) - V_{cb}V_{ub}^* (p_0 + \tfrac{1}{2}p_1\epsilon), \\ A(\bar{D}^0 \rightarrow \pi^+K^-) &= V_{cd}V_{us}^* (t_0 + \tfrac{1}{2}t_1\epsilon). \end{aligned} \quad (\text{A18})$$

Note that this is the most general decomposition, that is, all the subleading terms that we neglected can be absorbed into terms we kept. The decomposition in (A18) is equivalent to the decomposition given in (16)-(19), with the following translations: at  $\mathcal{O}(1/\epsilon)$ , and also including the  $\mathcal{O}(1)$  non-contraction term in  $p_0$ ,

$$p_0 = P + \tfrac{1}{2}T; \quad (\text{A19})$$

at  $\mathcal{O}(1)$ , and also including the  $\mathcal{O}(\epsilon)$  non-contraction term in  $s_1$ ,

$$p_1\epsilon = P\epsilon_P, \quad t_0 = T, \quad s_1\epsilon = P_{\text{break}} + \tfrac{1}{2}\epsilon_{T1}T; \quad (\text{A20})$$

and at  $\mathcal{O}(\epsilon)$

$$t_1\epsilon = T\epsilon_{T2}, \quad t_2\epsilon^2 = -P_{\text{break}}\epsilon_{sd}^{(2)}. \quad (\text{A21})$$

Fitting (A18) to the four branching ratios, dropping the  $V_{cb}V_{ub}^*$  terms, and taking  $\epsilon \in [0, 0.4]$  yields the fit for  $s_1\epsilon$  vs.  $t_0$  shown in Fig. 7. It is similar to the fit for  $P_{\text{break}}$  vs.  $T$  in Fig. 1, as one would expect, and confirms that  $s_1\epsilon \sim t_0$  and  $\epsilon' \sim \epsilon$ .

## Appendix B: Diagrammatical representation

Finally, we provide a derivation of the U-spin decomposition presented in (16)-(19) in terms of the diagrammatical representation. This will simplify the comparison with the result of Ref. [26], and provides further insight into the origin of the scaling  $\epsilon' \sim \epsilon_U$ .

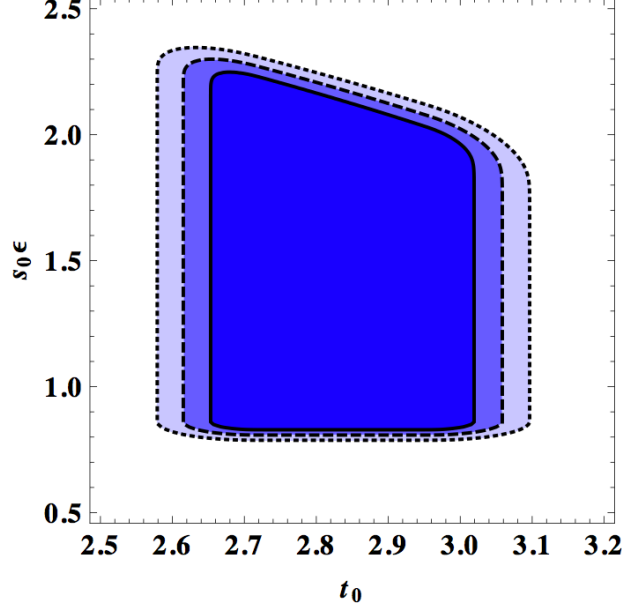


FIG. 7: The results of the fit of the U-spin decomposition in (A18) to the  $D \rightarrow K^+\pi^-$ ,  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $\pi^+K^-$  branching ratios. The contours denote the regions allowed at  $1\sigma$  (solid),  $2\sigma$  (dashed),  $3\sigma$  (dotted).

The  $\bar{D}^0$  decay amplitudes  $\bar{A}_f$  are split into “tree-level” amplitudes  $\bar{A}_f^T$  and penguin amplitudes  $\bar{A}_f^P$ . The former are sums of a “tree” diagram  $T$  and an “exchange” diagram  $E$ , and are given in full generality by

$$\begin{aligned}
\bar{A}_{K^+\pi^-}^T &= V_{cs}V_{ud}^*(T_{K\pi} + E_{K\pi}), \\
\bar{A}_{\pi^+\pi^-}^T &= -\frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*)(T_{\pi\pi} + E_{\pi\pi}), \\
\bar{A}_{K^+K^-}^T &= \frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*)(T_{KK} + E_{KK}), \\
\bar{A}_{K^-\pi^+}^T &= V_{cd}V_{us}^*(T_{\pi K} + E_{\pi K}).
\end{aligned} \tag{B1}$$

The  $T$  amplitudes are those with a  $u$  spectator quark, and the  $E$  amplitudes are the annihilation topology diagrams (cf. Fig. 8), in which the initial  $c\bar{u}$  quark pair annihilates into  $\bar{s}d$ ,  $\bar{s}s$ ,  $\bar{d}d$ , and  $\bar{d}s$  quark pairs, respectively.

In SCS decays we can divide  $E$  and  $T$  into two contributions: those that do not involve the  $\bar{s}s$  or  $\bar{d}d$  penguin contractions, denoted  $T_f$  and  $E_f$ , and those which do, denoted  $P_f^T$  and  $P_f^E$  and shown in the last two diagrams of Fig. 8. The penguin contractions arise from  $Q_{1,2}^{\bar{s}s} - Q_{1,2}^{\bar{d}d}$  and are U-spin violating. At the quark level they correspond to the difference between the rescattering contributions of the  $\bar{s}s$  and  $\bar{d}d$  quark pairs for a given final state.

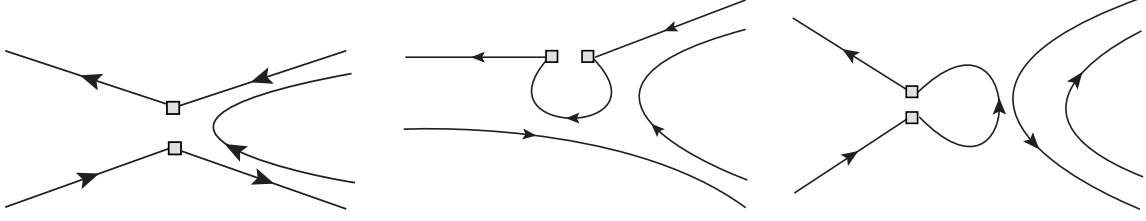


FIG. 8: From left to right: tree level exchange topology diagram,  $E_f$ , and the penguin contraction diagrams  $P_f^T$  and  $P_f^E$ .

We thus have

$$\begin{aligned} T_{KK} &= T_{KK}^s - (P_{KK}^{T,d} - P_{KK}^{T,s}), & T_{\pi\pi} &= T_{\pi\pi}^d + (P_{\pi\pi}^{T,d} - P_{\pi\pi}^{T,s}), \\ E_{KK} &= E_{KK}^s - (P_{KK}^{E,d} - P_{KK}^{E,s}), & E_{\pi\pi} &= E_{\pi\pi}^d + (P_{\pi\pi}^{E,d} - P_{\pi\pi}^{E,s}), \end{aligned} \quad (\text{B2})$$

where the superscripts  $s$  and  $d$  denote the identity of the relevant operators  $Q_{1,2}^{\bar{s}s}$  or  $Q_{1,2}^{\bar{d}d}$  (or the identity of the contracted quark pair in the penguin contractions). In the CF and DCS decays  $T$  and  $E$  of course do not receive contributions from the penguin contractions.

Formally, the  $P^T$  contain leading power as well as power correction contributions, while the  $P^E$  are pure power corrections. The scheme-dependent coefficients and leading  $\log \mu$  scale dependence entering the penguin contraction matrix elements cancels in the differences  $P_f^{T,s} - P_f^{T,d}$  and  $P_f^{E,s} - P_f^{E,d}$ .

In the penguin amplitudes the penguin contractions correspond to the sum of the rescattering contributions of the  $\bar{s}s$  and  $\bar{d}d$  quark pairs in  $(Q_{1,2}^s + Q_{1,2}^d)/2$ . Including the amplitudes for the penguin operators  $Q_{3,\dots,6}$ ,  $Q_{8g}$ , denoted  $\tilde{P}$ , and the above contracted and non-contracted amplitudes, we have

$$A_f^P = -V_{cb}V_{ub}^* (\tilde{P}_f + [P_f^{T,s} + P_f^{T,d}]/2 + [P_f^{E,s} + P_f^{E,d}]/2 + [T_f^q + E_f^q]/2), \quad (\text{B3})$$

where  $q = s(d)$  for  $f = K^+K^- (\pi^+\pi^-)$ . The scheme and scale dependence in the penguin contractions is now canceled by  $\tilde{P}_f$ . However, since the contributions of the  $\tilde{P}_f$  to the direct CP asymmetries are much smaller than observed, they are neglected in our fits.

To simplify the comparison with the previous section and with Section III, let us define the total non-contracted amplitudes,

$$\begin{aligned} \mathcal{T}_{K\pi} &\equiv T_{K\pi} + E_{K\pi}, & \mathcal{T}_{\pi K} &\equiv T_{\pi K} + E_{\pi K} \\ \mathcal{T}_{\pi\pi} &\equiv T_{\pi\pi}^d + E_{\pi\pi}^d, & \mathcal{T}_{KK} &\equiv T_{KK}^s + E_{KK}^s, \end{aligned} \quad (\text{B4})$$

and the total penguin contraction contributions in the tree and penguin amplitudes, ( $f = KK$  ( $\pi\pi$ ) for the  $K^+K^-$  ( $\pi^+\pi^-$ ) final state)

$$\mathcal{P}_f^t \equiv P_f^{T,d} - P_f^{T,s} + P_f^{E,d} - P_f^{E,s}, \quad \mathcal{P}_f^p \equiv (P_f^{T,d} + P_f^{T,s} + P_f^{E,d} + P_f^{E,s})/2. \quad (\text{B5})$$

In the latter it is understood that scale and scheme dependence has been subtracted out by  $\tilde{P}_f$ . We can now write the decay amplitudes in terms of the above quantities as

$$\begin{aligned} A(\bar{D}^0 \rightarrow K^+\pi^-) &= V_{cs}V_{ud}^* \mathcal{T}_{K\pi}, & A(\bar{D}^0 \rightarrow \pi^+K^-) &= V_{cd}V_{us}^* \mathcal{T}_{\pi K} \\ A(\bar{D}^0 \rightarrow \pi^+\pi^-) &= -\frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) (\mathcal{T}_{\pi\pi} + \mathcal{P}_{\pi\pi}^t) - V_{cb}V_{ub}^* \left( \tilde{P}_{\pi\pi} + \mathcal{P}_{\pi\pi}^p + \mathcal{T}_{\pi\pi}/2 \right), \\ A(\bar{D}^0 \rightarrow K^+K^-) &= \frac{1}{2}(V_{cs}V_{us}^* - V_{cd}V_{ud}^*) (\mathcal{T}_{KK} - \mathcal{P}_{KK}^t) - V_{cb}V_{ub}^* \left( \tilde{P}_{KK} + \mathcal{P}_{KK}^p + \mathcal{T}_{KK}/2 \right). \end{aligned} \quad (\text{B6})$$

In the U-spin limit, the non-contracted amplitudes satisfy

$$\mathcal{T}_{KK}^s = \mathcal{T}_{\pi\pi}^d = \mathcal{T}_{K\pi} = \mathcal{T}_{\pi K} \equiv T, \quad (\text{B7})$$

where  $T$  is defined in (11)-(12) in terms of operator matrix elements. Introducing the U-spin breaking parameters  $\epsilon_{T1}$  and  $\epsilon_{T2}$ , we can express these amplitudes at  $\mathcal{O}(\epsilon_U)$  as

$$\begin{aligned} \mathcal{T}_{KK}^s &= T(1 - \frac{1}{2}\epsilon_{T1}), & \mathcal{T}_{\pi\pi}^d &= T(1 + \frac{1}{2}\epsilon_{T1}) \\ \mathcal{T}_{K\pi} &= T(1 - \frac{1}{2}\epsilon_{T2}), & \mathcal{T}_{\pi K} &= T(1 + \frac{1}{2}\epsilon_{T2}). \end{aligned} \quad (\text{B8})$$

Similarly, in the U-spin limit, the penguin contractions in the penguin amplitudes satisfy

$$\mathcal{P}_{KK}^p = \mathcal{P}_{\pi\pi}^p \equiv P, \quad (\text{B9})$$

where  $P$  is defined in (10). Introducing the U-spin breaking parameter  $\epsilon_P$ , we can write them at  $\mathcal{O}(\epsilon_U)$  as

$$\mathcal{P}_{KK}^p = P(1 + \frac{1}{2}\epsilon_P), \quad \mathcal{P}_{\pi\pi}^p = P(1 - \frac{1}{2}\epsilon_P). \quad (\text{B10})$$

While the  $\mathcal{P}_f^t$  vanish in the U-spin limit, at  $\mathcal{O}(\epsilon_U)$  we have

$$\mathcal{P}_{KK}^t = \mathcal{P}_{\pi\pi}^t \equiv P_{\text{break}}, \quad (\text{B11})$$

where  $P_{\text{break}}$  is defined in terms of operator matrix elements in (13).  $P_{\text{break}}$  and  $P$  can be related by

$$P_{\text{break}} = \epsilon_{sd}^{(1)} P, \quad (\text{B12})$$

where the U-spin breaking parameter  $\epsilon_{sd}^{(1)}$  accounts for the difference in  $\bar{s}s$  and  $\bar{d}d$  rescattering contributions at  $\mathcal{O}(\epsilon_U)$ . The difference between  $\mathcal{P}_{KK}^t$  and  $\mathcal{P}_{\pi\pi}^t$  enters formally at  $\epsilon_U^2$ . Introducing a new U-spin breaking parameter  $\epsilon_{sd}^{(2)}$  to take this difference into account yields the relations

$$\mathcal{P}_{KK}^t = P_{\text{break}}(1 + \tfrac{1}{2}\epsilon_{sd}^{(2)}), \quad \mathcal{P}_{\pi\pi}^t = P_{\text{break}}(1 - \tfrac{1}{2}\epsilon_{sd}^{(2)}). \quad (\text{B13})$$

Finally, substituting above expressions (B8), (B10), (B13) for  $\mathcal{T}_f$ ,  $\mathcal{P}_f^p$ ,  $\mathcal{P}_f^t$ , respectively, in (B6) and neglecting the  $\tilde{P}_f$ , yields the diagrammatic expressions for the decay amplitudes given in (16)-(19) of Section III. We have assumed that the penguin contractions are dynamically enhanced by  $\mathcal{O}(1/\epsilon')$ , where  $\epsilon' \sim \epsilon_U$  which leads to the scalings for the various amplitude contributions given in (20) and (21).

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